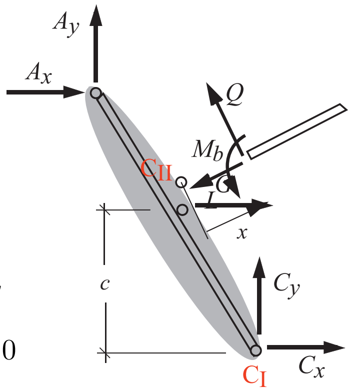
Musterlösung Mechanik I F14

Aufgabe S1 F14



Aa) FreischnittE I und II bzw Gesamtsystem aus I und II:

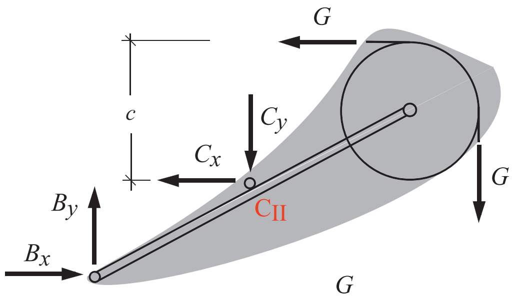
(1) $\sum M_{(C_I)} = 0 \qquad A_x\,(a+c) + A_y\,b + G\,c = 0$

$$\text{mit} \quad c = b\,\tan\alpha + r \quad \text{und} \quad G = m_5 g$$

(2) $\sum M_{(C_{II})} = 0 \qquad B_y\,b - B_x\,b\tan\alpha + G\,(b+r) - G\,c = 0$

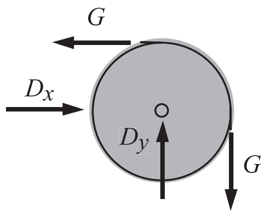
(3) $\sum F_y = 0 \qquad A_y + B_y = G$

(4) $\sum F_x = 0 \qquad A_x + B_x = 0$



Vier Gleichungen für vier Unbekannte: $A_x\,,\ B_x\,,\ A_y\,,\ B_y$

Ab) Freischnitte Balken 1 und Rolle:

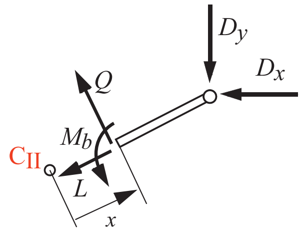


$$\sum F_x = 0 \qquad -C_x + B_x - G = 0$$

$$\sum F_y = 0 \qquad -C_y + B_y - G = 0$$

$$\sum F_x = 0 \qquad D_x - G = 0$$

$$\sum F_y = 0 \qquad D_y - G = 0$$

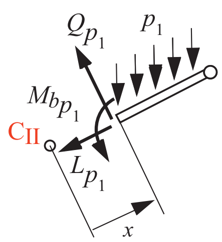


Ac) Schnittreaktionen

$$\sum F_x = 0 \qquad L(x) = -D_x\,\cos\alpha - D_y\,\sin\alpha$$

$$\sum F_y = 0 \qquad Q(x) = -D_x\,\sin\alpha + D_y\,\cos\alpha$$

$$\sum M = 0 \qquad M_b(x) = (-D_x\,\sin\alpha + D_y\,\cos\alpha)\,\left(\frac{b}{\cos\alpha} - x\right)$$



B)

$$\sum M = 0 \qquad M_{b_{p_1}}(x) = -p_1\left(\frac{b}{\cos\alpha} - x\right)^2/2 \quad \text{mit} \quad p_1 = \frac{G_1\,\cos\alpha}{2b}$$

Aufgabe F1 F14 (17 Punkte)

a)

Druckbelastung in z-Richtung mit „Actio = reactio“ : $\sigma_{z1} = \sigma_{z2} = \sigma_z = -F/(4a^2)$

Dehnung in x, y-Richtung Körper 1 : $\varepsilon_{x1} = \varepsilon_{y1} = 0\,,\ \varepsilon_{z_1} = \Delta h_1/h_1 = -\delta_1/h_1$

$$\varepsilon_{x1} = 0 = 1/E_1\Big(\sigma_{x1} - \nu_1\,(\sigma_{y1} + \sigma_z)\Big)$$
$$\varepsilon_{y1} = 0 = 1/E_1\Big(\sigma_{y1} - \nu_1\,(\sigma_z + \sigma_{x1})\Big)$$
$$\varepsilon_{z1} = -\delta_1/h_1 = 1/E_1\Big(\sigma_z - \nu_1\,(\sigma_{x1} + \sigma_{y1})\Big)$$

$$\sigma_{x1} = \sigma_{y1} = \nu_1\,\sigma_z/(1-\nu_1)\,,\ \delta_1/h_1 = -\sigma_z/E_1\Big(1 - \nu_1^2/(1-\nu_1)\Big)$$

$$\Rightarrow \quad F = \frac{4a^2\,E_1\,\delta_1/h_1}{1 - 2\nu_1^2/(1-\nu_1)}$$

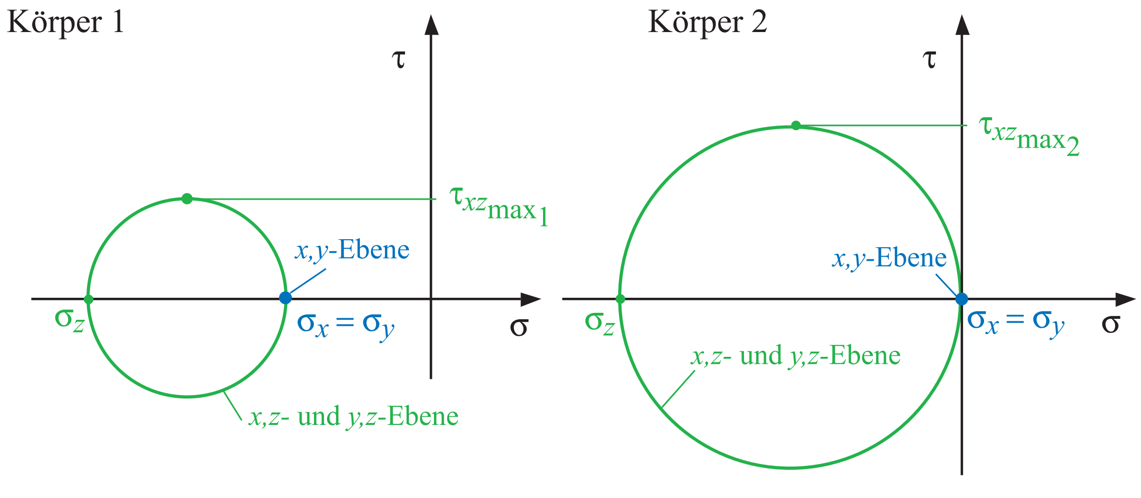
b,c) Eindimensionaler Spannungszustand im Körper 2 mit $\sigma_{x2} = \sigma_{y2} = 0$ und $\sigma_z = -F/(4a^2)$:

$$\varepsilon_{x2} = \varepsilon_{y2} = \Delta a/a = \delta_2/a = -\nu_2\,\sigma_z/E_2$$
$$\varepsilon_{z2} = \Delta h_2/h_2 = \sigma_z/E_2$$

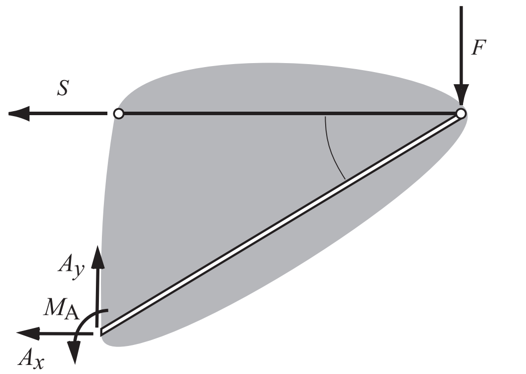
$$\Rightarrow \quad \delta_2 = \nu_2\,a\,F/(4a^2)/E_2\,,\ \Delta a = a\,\sigma_z/E_2 = -a\,F/(4a^2E_2)$$

d,e) Mohrscher Spannungskreis qualitativ für $\sigma_z < \sigma_x = \sigma_y = \nu/1 - \nu\sigma_z < 0$ bzw. $\sigma_y < \sigma_x = \sigma_y = 0$:

Abgelesen: $\tau_{xz_{\max 1}}/\tau_{xz_{\max 2}} = (\sigma_{x1} - \sigma_z)/(-\sigma_z) = (2\nu_1 - 1)/(\nu_1 - 1)$



Aufgabe F2 F14 (14 Punkte)



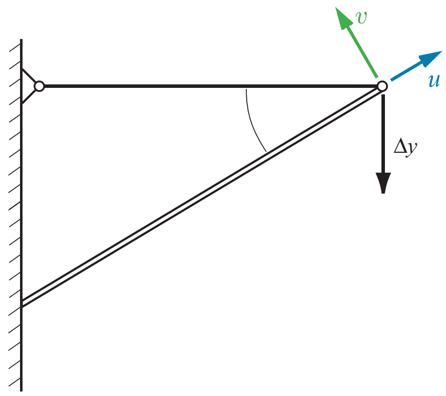
a) Freischnitt Gesamtsystem:

(1) $\sum M_{(A)} = 0 \qquad -M_A + F\,a\cos\alpha - S\,a\sin\alpha = 0$

(2) $\sum F_x = 0 \qquad -A_x - S = 0$

(3) $\sum F_y = 0 \qquad A_y - F = 0$

(Drei Gleichungen für vier Unbekannte: A_x, A_y, M_A, S)



b) Verschiebeplan, Balkenbiegung und Hookesches Gesetz:

$$
\begin{aligned}
(4) \quad \Delta l_2 &= \cancelto{0}{u}\cos\alpha - v\,\sin\alpha \\
(5) \quad v &= \frac{(-F\,\cos\alpha + S\,\sin\alpha)\,a^3}{3B} \\
(6) \quad \Delta l_2 &= \frac{S\,l_2}{D} \quad \text{mit} \quad l_2 = a\,\cos\alpha \\
(7) \quad \Delta y &= -\cancelto{0}{u}\sin\alpha + v\,\cos\alpha
\end{aligned}
$$

7 Gleichungen für 7 Unbekannte: A_x, A_y, M_A, S, Δl_2, v, Δy