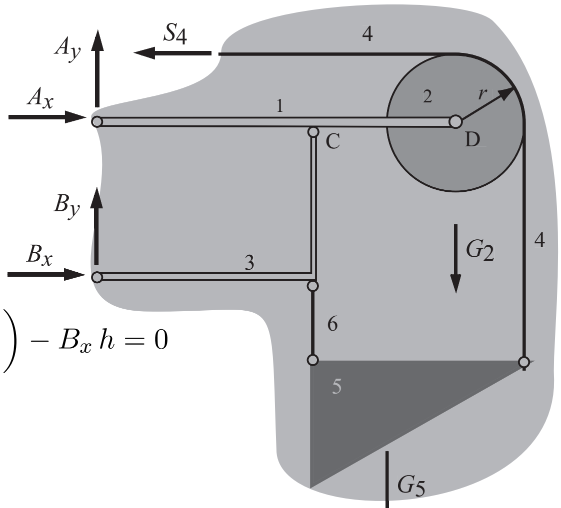
Musterlösung Mechanik I F15

Aufgabe S1 F15

Aa) Freischnitte Scheibe, Gesamtsystem und Balken 3:



(1) $\quad \sum M_{(\mathrm{I})} = 0 \qquad S_4\, b - G_5\, \dfrac{b}{3} = 0 \quad \Rightarrow \quad S_4 = \dfrac{1}{3}\, G_5$

(2) $\quad \sum M_{(\mathrm{II})} = 0 \qquad S_6\, b - G_5\, \dfrac{2\,b}{3} = 0 \quad \Rightarrow \quad S_6 = \dfrac{2}{3}\, G_5$

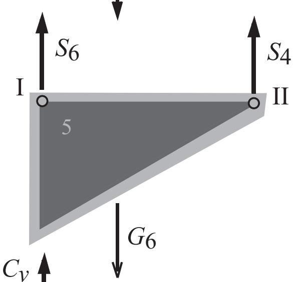
(3) $\quad \sum M_{(\mathrm{A})} = 0 \qquad -S_4\, r + G_2\, (a + b - r) + G_5 \left(a + \dfrac{b}{3}\right) - B_x\, h = 0$

$\Rightarrow \quad B_x$

(4) $\quad \sum F_x = 0 \qquad A_x + B_x - S_4 = 0 \quad \Rightarrow \quad A_x$

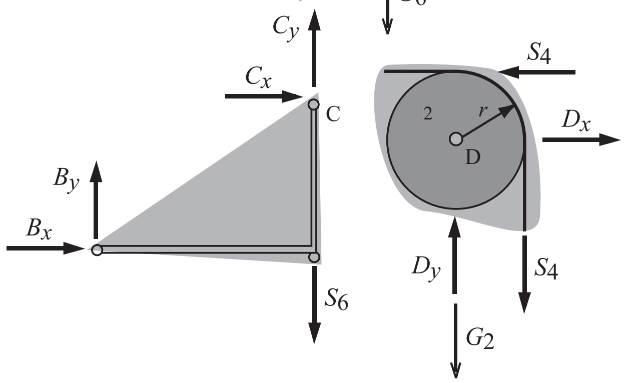
(5) $\quad \sum M_{(\mathrm{C})} = 0 \qquad B_y\, a - B_x\, h = 0 \quad \Rightarrow \quad B_y$

(6) $\quad \sum F_y = 0 \qquad A_y + B_y - G_2 - G_5 = 0 \quad \Rightarrow \quad A_y$



Sechs Gleichungen für sechs Unbekannte: $A_x\,,\ B_x\,,\ A_y\,,\ B_y\,,\ S_4\,,\ S_6$

Ab) Freischnitt Balken 3:



(7) $\quad \sum F_x = 0 \qquad C_x + B_x = 0$

(8) $\quad \sum F_y = 0 \qquad C_y + B_y - S_6 = 0$

Weitere zwei Gleichungen für Unbekannte: $C_x\,,\ C_x$

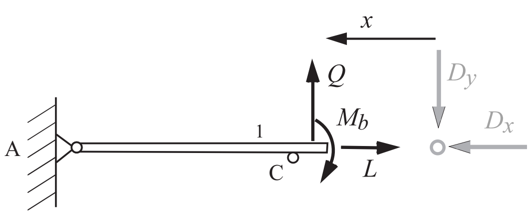
Freischnitt Rolle:

(9) $\quad \sum F_x = 0 \qquad D_x - S_4 = 0$

(10) $\quad \sum F_y = 0 \qquad D_y - S_4 - G_2 = 0$

Weitere zwei Gleichungen für Unbekannte: $D_x\,,\ D_x$

Ac) Schnittreaktionen für $0 \leq x \leq d$:



$$\sum F_x = 0: \quad L(x) = -D_x$$

$$\sum F_y = 0: \quad Q(x) = -D_y$$

$$\sum M = 0: \quad M_b(x) = D_y\, x$$

B)

$$\sum M = 0: \quad M_{b_{p_1}}(x) = -p_1\, x\, \frac{x}{2} \quad \text{mit} \quad p_1 = \frac{G_1}{a + b - r}$$

Aufgabe F1 F15

a) Verformungsaussage: $c_1 + \Delta c_1 + c_2 + \Delta c_2 = c \quad \Rightarrow \quad \Delta c_1 + \Delta c_2 = c - (c_1 + c_2) = -\Delta c \qquad (*)$

Wärmeausdehnung: $\Delta c_1 = c_1\, \alpha_1\, \Delta T\,, \quad \Delta c_2 = c_2\, \alpha_2\, \Delta T$

$$\Rightarrow \quad \Delta T = \frac{-\Delta c}{c_1\, \alpha_1 + c_2\, \alpha_2} \quad (< 0)$$

b) Nebenbedingungen:

$$\sigma_{1,x} = \sigma_{1,y} = \sigma_{2,x} = \sigma_{2,y} = 0\,, \qquad \sigma_{1,z} = \sigma_{2,z} \overset{!}{=} \sigma_z$$

Hookesches Gesetz:

$$\begin{aligned}
\varepsilon_{1,x} &= \frac{1}{E_1}\left(\sigma_x - \nu_1\left(\sigma_y + \sigma_z\right)\right) &=& -\frac{\nu_1}{E_1}\,\sigma_z \\
\varepsilon_{1,y} &= \frac{1}{E_1}\left(\sigma_y - \nu_1\left(\sigma_z + \sigma_x\right)\right) &=& -\frac{\nu_1}{E_1}\,\sigma_z = \varepsilon_{1,x} \\
\varepsilon_{1,z} &= \frac{1}{E_1}\left(\sigma_z - \nu_1\left(\sigma_x + \sigma_y\right)\right) &=& -\frac{1}{E_1}\,\sigma_z = \frac{\Delta c_1}{c_1}
\end{aligned}$$

Analog für Quader 2: $\varepsilon_{2,x} = \varepsilon_{2,y} = -\frac{\nu_2}{E_2}\,\sigma_z\,, \quad \varepsilon_{2,z} = -\frac{1}{E_2}\,\sigma_z = \frac{\Delta c_2}{c_2}$

Aus

$$\left.\begin{aligned}
\Delta c_1 &= \sigma_z\,\frac{c_1}{E_1} \\
\Delta c_2 &= \sigma_z\,\frac{c_2}{E_2}
\end{aligned}\right\} \quad \text{folgt mit Gl.(*)} \qquad \sigma_z = -\frac{\Delta c}{c_1/E_1 + c_2/E_2} \quad (< 0)$$

c)

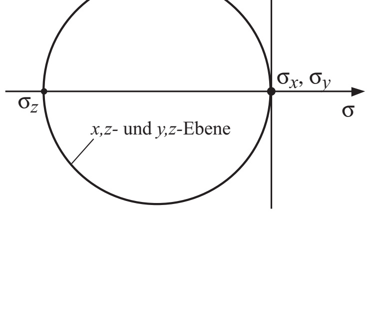
$$\begin{aligned}
\varepsilon_{1,x} = -\frac{\nu_1}{E_1}\,\sigma_z = \frac{\Delta a_1}{a} \quad &\Rightarrow \quad \Delta a_1 = \frac{\nu_1}{E_1}\,\frac{\Delta c}{c_1/E_1 + c_2/E_2} \\
\text{analog} \quad \varepsilon_{2,x} = -\frac{\nu_2}{E_2}\,\sigma_z = \frac{\Delta a_2}{a} \quad &\Rightarrow \quad \Delta a_2 = \frac{\nu_2}{E_2}\,\frac{\Delta c}{c_1/E_1 + c_2/E_2}
\end{aligned}$$

mit $\Delta a = \max(\Delta a_1, \Delta a_2)$

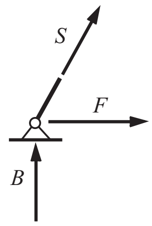
Analog $\Delta b = \max(\Delta b_1, \Delta b_2)$ mit

$$\begin{aligned}
\varepsilon_{1,y} = -\frac{\nu_1}{E_1}\,\sigma_z = \frac{\Delta b_1}{b} \quad &\Rightarrow \quad \Delta b_1 = \frac{\nu_1}{E_1}\,\frac{\Delta c}{c_1/E_1 + c_2/E_2} \\
\text{analog} \quad \varepsilon_{2,y} = -\frac{\nu_2}{E_2}\,\sigma_z = \frac{\Delta b_2}{b} \quad &\Rightarrow \quad \Delta b_2 = \frac{\nu_2}{E_2}\,\frac{\Delta c}{c_1/E_1 + c_2/E_2}
\end{aligned}$$

Abgelesen: $T_{\max} = \dfrac{-\sigma_z}{2}$ und $\varphi(\tau_{\max}) = \dfrac{\pi}{4}$

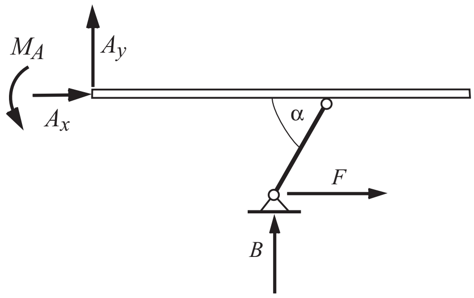


Aufgabe F2 F15



a) Freischnitt Lager bei B:

$$\sum F_x = 0 \quad S\,\cos\alpha + F = 0 \quad \Rightarrow \quad S = -\frac{F}{\cos\alpha}\ (< 0\surd)$$



b) Am Freischnitt Lager B:

$$\sum F_y = 0 \quad S\,\sin\alpha + B = 0 \quad \Rightarrow \quad B = -S\,\sin\alpha\ (> 0\surd)$$

Freischnitt Gesamtsystem:

$$\sum F_x = 0 \qquad A_x + F = 0 \qquad \Rightarrow \quad A_x = -F$$

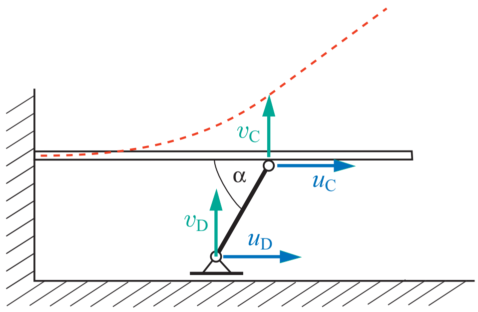
$$\sum M_{(\mathrm{A})} = 0 \qquad M_A + B\,(a - \frac{h}{\tan\alpha}) + F\,h = 0 \qquad \Rightarrow \quad M_A = -S\,a\,\sin\alpha\ (< 0\surd)$$

$$\sum F_y = 0 \qquad A_y + B = 0 \qquad \Rightarrow \quad A_y = -F\,\tan\alpha$$

c) Verschiebeplan des Stabes:

$$\begin{aligned} \Delta l_2 &= u_{\mathrm{C}}\,\cos\alpha + v_{\mathrm{C}}\,\sin\alpha - u_{\mathrm{D}}\,\cos\alpha - v_{\mathrm{d}}\,\sin\alpha \\ u_{\mathrm{C}} &\approx 0 \\ v_{\mathrm{D}} &= 0 \end{aligned}$$

$$\Rightarrow \quad (1) \quad \Delta l_2 = v_{\mathrm{C}}\,\sin\alpha - u_{\mathrm{D}}\,\cos\alpha$$



Balkenbiegung:

$$(2) \quad v_{\mathrm{C}} = -\frac{S\,\sin\alpha\,a^3}{3\,IE}\ (> 0\surd)$$

Hookesches Gesetz:

$$(3) \quad \Delta l_2 = \frac{S\,l_2}{EA}\ (< 0\surd) \quad \text{mit} \quad l_2 = \frac{h}{\tan\alpha}$$

Drei Gleichugen für drei Unbekannte: $\Delta l_2\,,\ v_{\mathrm{C}}\,,\ u_{\mathrm{D}} \qquad \Rightarrow \quad u_{\mathrm{D}}$